

Vortex solitons in dispersive nonlinear Kerr type media

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Abstract

We have investigated the nonlinear amplitude vector equation governing the evolution of optical pulses in optical and UV region. We are normalizing this equation for the cases of different and equal transverse and longitudinal size of optical pulses or modulated optical waves, of weak and strong dispersion. This gives us the possibility to reduce the amplitude equation to different nonlinear evolution equations in the partial cases. For some of these nonlinear equations exact vortex solutions are found. Conditions for experimental observations of these vortices are determined. We have found large spectral zones on the 'wings' of the electron resonances of metal vapors and also near to the plasma frequency where the amplitude equation can be reduced to 3D+1 vector nonlinear Schrödinger equation.

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1 Introduction

The investigation of optical pulses and of modulated periodical waves in wave guide structures as fibers and planar waveguides is connected with the additional requirements to the optical property of the materials. One standard way for confinement the optical beams and pulses is by variation of the linear or nonlinear refractive index in plane, which is transverse to the propagation direction. In an isotropic dispersive media the refractive index depends only on the frequency, the group velocity depends on the chromatic dispersion and the wave guide modes can not be introduced in linear regime of propagation. To provide one correct analysis in this case we need to get in mind also the different spatial size of the optical pulses. The longitudinal spatial size is determined from the time duration by the relations $z_0 = v_g t_0 = ct_0/n_g$, where a typical value of the group index n_g in transparency region of pure silica is ($n_g \cong 1.5$). For example, the typical transverse size of laser pulses is evaluated from $r_\perp = 1 - 5 \text{ mm}$ to $r_\perp = 100 \text{ }\mu\text{m}$. While for the longitudinal size of pulses or spatial period of modulated waves, there are mainly two possibilities: a. Pulses from nanoseconds up to 40 – 100 picoseconds. For such pulses $z_0 \gg r_\perp$ and their shape is more close to optical filaments. b. Pulses from few picoseconds to 100 femtoseconds. In this case $z_0 \sim r_\perp$ and the pulses look like as optical bullets. When the transverse and longitudinal part are approximately equal (case b), the ignoring of the second derivative in z direction in amplitude equation is not possible. The propagation of such type of short optical pulses in nonlinear media must be characterized, not only with the equal dimensions in x , y and z direction, but also with non-stationary optical linear and nonlinear response of the media. That is why we can expect, that the equations governing the propagation of such type of pulses, like optical bullets, are different from the well known paraxial approximated equations, which describe evolution of nanosecond and picosecond pulses [1, 2, 3]. Another basic characteristic of the pulses is their polarization. As it is well known, in the case of plane monochromatic wave, the electromagnetic field is polarized in the plane perpendicular to the direction of propagation. In this case four Stokes parameters, two amplitudes and two phases, connected with the transverse components of the electromagnetic field can be introduced [4]. This corresponds to SU(2) symmetry of propagation of the monochromatic field. The nanosecond and hundred picosecond optical pulses admit very little spectral bandwidth, and the polarization component in the propagation direction is small in respect to the transverse components. With good

approximation these pulses or modulated waves also can be investigated as polarized in x, y plane. Their spatial form also is more close to the cw wave, as the longitudinal size is hundred times greater, than the transverse size. In contrast to the case of long pulses, the few ps and femtosecond optical waves are with relatively equal size in x, y, z direction and admit large spectral width. For them such coherent SU(2) tensor can not be introduced. The experimental results show, that the electric field of sub-picosecond and femtosecond pulses, always contains also third longitudinal component. With the enlargement of the spectral width of the vector field also the longitudinal component grows (the component normal to the standard Stokes coherent polarization plane). Recently Carozzi et al. have shown that for the wave packets higher class of symmetry - SU(3) exists [5]. Using the presentation of Gell-Mann of SU(3), they prove that five independent parameters, three amplitudes and two phases, define the dynamics of propagation of the vector field. This corresponds to a three-component vector field. The above made analysis and the experimental results show that in the investigation of sub-picosecond and femtosecond pulses we must always have in mind the three-component vector character of the electromagnetic field. An initial investigation of the propagation of the optical pulses was provided in the frame of spatiotemporal scalar evolution equation [12, 13]. We found that vector generalization of such spatiotemporal model can be applied to optical pulses or to modulated periodical waves in UV region for dielectrics with strong dispersion.

2 Nonlinear amplitude vector equation

Starting from the Maxwell equation for an isotropic, dispersive, nonlinear Kerr-type media with no stationary response, we derive the next slowly varying vector amplitude equation [16]:

$$i \left(\frac{\partial \vec{A}}{\partial t} + v \frac{\partial \vec{A}}{\partial z} + \left(n_2 + \frac{k_0 v}{2} \frac{\partial n_2}{\partial \omega} \right) \frac{\partial \left(|\vec{A}|^2 \vec{A} \right)}{\partial t} \right) =$$

$$\frac{v}{2k_0} \Delta \vec{A} - \frac{v}{2} \left(k_0'' + \frac{1}{k_0 v^2} \right) \frac{\partial^2 \vec{A}}{\partial t^2} + \frac{n_2 k_0 v}{2} |\vec{A}|^2 \vec{A},$$

where k_0 , v , k_0'' and n_2 are the carrying wave number, group velocity, dispersion of the group velocity, and the nonlinear refractive index respectively. With \vec{A} we denote the slowly varying vector amplitude of the electrical field. The equation (1) is written in second approximation to the linear dispersion and in first approximation to the nonlinear dispersion. The kind of nonlinearity is connected with the fact, that in this paper, as in the previous one [16], we investigate only linearly polarized electrical field. The dependance of the nonlinear polarization from different kinds of polarizations of the electrical field is discussed in the Appendix 1. We will apply to the basic amplitude equation (1) the method of different and equal transverse and longitudinal spatial scales, as it is connected with the natural initial shape of the optical pulses. This application is one simplification of the multi-scale method, introduced in the nonlinear optics in [6, 7, 8] and also in the hydrodynamics [9].

3 Case of different transverse and longitudinal size

Applying a "moving in time" ($t' = t - z/v$; $z' = z$) transformation to the vector amplitude equation (1), we obtain:

$$\begin{aligned}
& -i \left(\frac{\partial \vec{A}}{\partial z'} + \frac{1}{v} \left(n_2 + \frac{k_0 v}{2} \frac{\partial n_2}{\partial \omega} \right) \frac{\partial \left(|\vec{A}|^2 \vec{A} \right)}{\partial t'} \right) + \frac{1}{2k_0} \Delta_{\perp} \vec{A} - \frac{k_0''}{2} \frac{\partial^2 \vec{A}}{\partial t'^2} - \\
& \frac{1}{2k_0} \left(\frac{\partial^2 \vec{A}}{\partial z'^2} - \frac{2}{v} \frac{\partial^2 \vec{A}}{\partial t' \partial z'} \right) + \frac{n_2 k_0 v}{2} |\vec{A}|^2 \vec{A} = 0,
\end{aligned} \tag{2}$$

where $\Delta_{\perp} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. The estimation of the influence of different terms in equation (2) can be reached by writing it in dimensionless variables. To make one assessment of the difference between transverse and longitudinal dimension of the pulses we are introducing separately transverse r_{\perp} and longitudinal z_0 constants. Defining the rescaled variables:

$$\vec{A} = A_0 \vec{A}'; \quad x = r_{\perp} x'; \quad y = r_{\perp} y'; \quad z' = z_0 z''; \quad t' = t_0 t'', \tag{3}$$

and constants:

$$z_0/t_0 = v; \quad \alpha = k_0 z_0; \quad \delta = r_\perp/z_0; \quad z_{dis} = t_0^2/k''; \quad (4)$$

$$\beta_1 = k_0 r_\perp^2/z_{dis}; \quad \gamma = k_0^2 r_\perp^2 n_2 |A_0|^2/2; \quad \gamma_1 = n_2 |A_0|^2/2,$$

equation (2) can be transformed in the following (the primes and the seconds are not written for clarity):

$$\begin{aligned} & -i\alpha\delta^2 \left(\frac{\partial \vec{A}}{\partial z} + \gamma_1 \frac{\partial \left(|\vec{A}|^2 \vec{A} \right)}{\partial t} \right) + \frac{1}{2} \Delta_\perp \vec{A} - \frac{1}{2} \beta_1 \frac{\partial^2 \vec{A}}{\partial t^2} - \\ & \frac{1}{2} \delta^2 \left(\frac{\partial^2 \vec{A}}{\partial z^2} - 2 \frac{\partial^2 \vec{A}}{\partial t \partial z} \right) + \gamma |\vec{A}|^2 \vec{A} = 0, \end{aligned} \quad (5)$$

where $z_{dis} = t_0^2/k''$ and in the transparency region we use the approximation $\partial n_2/\partial \omega \simeq 0$.

3.1 Pulse propagation in optical region: weak dispersion

In the optical region the wave vector is valued from $k_0 \sim 10^4$ to $k_0 \sim 10^5 \text{ cm}^{-1}$. The typical value of the dispersion parameter in the transparency optical region of dielectrics is from $k'' \sim 10^{-28} \text{ s}^2/\text{cm}$ up to $k'' \sim 10^{-24} \text{ s}^2/\text{cm}$ in UV region [17]. The transfer sizes r_\perp of laser generated optical pulses or modulated periodical in time waves are valued from $r_\perp < 1$ to $r_\perp \sim 10^{-2} \text{ cm}$. We will investigate laser pulses or modulated periodical waves with time duration or modulation period from few nanoseconds ($t_0 \sim 10^{-9} \text{ s}$) up to 40 – 100 picoseconds ($t_0 \sim 4 - 10 \times 10^{-11} \text{ s}$) in the transparency region of a nonlinear Kerr type media. The group velocity index in this case is about $n_g \simeq 1.5$ and using the relations $z_0 = v_g t_0 = c t_0/n_g \sim 10^1 - 10^2 \text{ cm}$, we find that the dimensionless parameter in front of second derivative in z direction and cross-term is very small $\delta^2 = r_\perp^2/z_0^2 \simeq 10^{-2} - 10^{-4}$. Another important relation is that if slowly-varying amplitudes are used $\alpha = k_0 z_0 \simeq 10^3 - 10^4$.

For such typical values of the dispersion k'' , wave vector k_0 , time duration t_0 and transverse dimension r_\perp of the optical wave, the dimensionless parameter β_1 in front of the dispersion term in equation (5) is very small $\beta = k_0 r_\perp^2 / z_{dis} \sim 10^{-3} - 10^{-4}$. Here we study the cases with power near the critical for self-focusing $\gamma \cong 1$. These valuations give us possibility to estimate dimensionless constant in front of the different terms of normalized amplitude equation (5) for the transparency optical region of a dispersive nonlinear Kerr type media:

$$\begin{aligned} \alpha = k_0 z_0 &\simeq 10^3 - 10^4; \quad \delta^2 = r_\perp^2 / z_0^2 \simeq 10^{-2} - 10^{-4}; \\ \beta_1 = k_0 r_\perp^2 / z_{dis} &\simeq 10^{-3} - 10^{-4}; \\ \gamma = k_0^2 r_\perp^2 n_2 |A_0|^2 &\simeq 1; \quad \gamma_1 = n_2 |A_0|^2 \ll 1. \end{aligned} \quad (6)$$

Neglecting the small terms in (5) we obtain the next amplitude equation:

$$-i \frac{\partial \vec{A}}{\partial z} + \frac{1}{2} \Delta_\perp \vec{A} + |\vec{A}|^2 \vec{A} = 0. \quad (7)$$

As it can be expected, the dynamics of such kind of long optical pulses is governed by the well known paraxial approximation. This equation was introduced initially for optical beam [1, 2, 3]. The fact, that with such type of nonlinear paraxial equation it is possible to investigate non-stationary processes is well known also [10, 11, 7]. It seems natural the dynamics of propagation of long pulses to be governed by the equation which describes dynamics of optical beam. The paraxial approximation is applicable even when the relation between the longitudinal and the transverse part is of the order of ten ($\delta = r_\perp / z_0 \sim 1/10$). The dimensionless coefficient in front of the term with second derivative in z direction and cross-term is of the order of square of δ ($\delta^2 \sim 1/100 \ll 1$), the term can be neglected and this leads again to the paraxial equation (7). Effects of self-steepening due to the first order of the nonlinear dispersion (second term in equation (5)) is not possible as always $\gamma_1 \approx n_2 |A|^2 \ll 1$. Different kind of generalization of equation (7) in respect to nonlinearities [12, 18, 19, 20] and multi-component vector fields [21, 22] was performed. It is important to point here that the second derivative in z direction and the crossed term are neglected from the difference between transverse and longitudinal size ($\delta^2 = r_\perp^2 / z_0^2 \simeq 10^{-2} - 10^{-4} \ll 1$), and not because the slowly varying amplitude is used. When we investigate the optical pulses with equal transverse and longitudinal size

$\delta^2 = r_{\perp}^2/z_0^2 \simeq 1$, the second derivative in z direction and cross term of the amplitude function are of the same order that the transverse Laplacian, and these terms influence considerably on the dynamics of propagation. This important case we will investigate in the next section. When the power of the pulse is less than the critical for self-focusing $\gamma \ll 1$ the diffraction term dominates and the propagation of the optical pulses is governed by the next linear paraxial equation:

$$-i\frac{\partial \vec{A}}{\partial z} + \frac{1}{2}\Delta_{\perp}\vec{A} = 0. \quad (8)$$

3.2 Optical pulses in UV region: strong dispersion. Vortex solutions

In UV region of media with high density as liquids and dielectrics the dispersion k'' increases considerably and can reach the values from $k'' \sim 10^{-24} - 10^{-25} \text{ s}^2/\text{cm}$. The carrying wavenumber is valued from $k_0 \sim 10^5 \text{ cm}^{-1}$ to $k_0 \sim 10^6 \text{ cm}^{-1}$ and in this spectral region we can reach value with equal diffraction and dispersion length, which corresponds to $\beta_1 = k_0 r_{\perp}^2/z_{dis} \sim 1$. We use for example bulk fused silica and calculate the linear refractive index from the Sellmeier relations [23]:

$$n^2 = 1 + \sum_{j=1}^3 \frac{B_j \omega_j}{\omega_j - \omega}, \quad (9)$$

with coefficients $B_1 = 0.6961663$, $B_2 = 0.4079426$, $B_3 = 0.897479$, wavelengths $\lambda_1 = 0.0684043 \text{ }\mu\text{m}$, $\lambda_2 = 0.1162414 \text{ }\mu\text{m}$, $\lambda_3 = 9.896161 \text{ }\mu\text{m}$ where $\lambda_j = 2\pi c/\omega_j$. The choice of carrying wavelength for the optical packet at $\lambda = 0.264 \text{ }\mu\text{m}$, ($\omega = 0.7131606 \cdot 10^{16} \text{ Hz}$) is connected with the facts, that fused silica is still transparent on this wavelength and it can be reached easy way by fourth harmonics from Nd YAG laser on $\lambda = 1.064 \text{ }\mu\text{m}$. Considering the case of pulses or modulated waves with intensity near the critical for self-focusing $\gamma \sim 1$, time duration $t_0 \simeq 30 - 40 \text{ ps}$ and using the relations $k(\omega) = \sqrt{\omega^2 n^2(\omega)/c^2}$, $k'' = \frac{\partial^2 k}{\partial \omega^2}$, the next values for laser characteristics and material constants are obtained:

$$k_0 = 2.378 \times 10^5 \text{ cm}^{-1}; \quad k'' = 1.99 \times 10^{-25} \text{ s}^2/\text{cm}; \quad n_g \sim 1.65;$$

$$\begin{aligned}
r_{\perp} &\simeq (1.4 - 1.6) \times 10^{-1} \text{ cm}; \quad z_{dis} \simeq (0.45 - 0.8) \times 10^4 \text{ cm}; \\
z_0 &\sim (6 - 8) \times 10^{-1} \text{ cm}; \quad k_0^2 r_{\perp}^2 \simeq (1.1 - 1.5) \times 10^9; \\
n_2 |A_0|^2 &\simeq (1.1 - 1.5) \times 10^{-9}.
\end{aligned} \tag{10}$$

These parameters correspond to the next dimensionless constant:

$$\begin{aligned}
\beta_1 &= k_0 r_{\perp}^2 / z_{dis} \simeq 1; \quad \alpha \cong 10^4; \\
\delta^2 &= r_{\perp}^2 / z_0^2 \simeq 10^{-2}; \quad \gamma = k_0^2 r_{\perp}^2 n_2 |A_0|^2 \simeq 1; \\
\alpha \delta^2 \gamma_1 &= \alpha \delta^2 n_2 |A_0|^2 \simeq 10^{-6} \ll 1.
\end{aligned} \tag{11}$$

Neglecting the small terms from the amplitude equation (5) we obtain the next equation governing the propagation of optical pulses with different transfer and longitudinal size in UV region of dielectrics:

$$-i\alpha\delta^2 \frac{\partial \vec{A}}{\partial z} + \frac{1}{2} \Delta_{\perp} \vec{A} - \frac{1}{2} \beta_1 \frac{\partial^2 \vec{A}}{\partial t^2} + \gamma |\vec{A}|^2 \vec{A} = 0. \tag{12}$$

The scalar variant of this spatiotemporal equation (12) was introduced for first time by Silberberg [12] and attracts a growing interest. In the beginning this equation was suggested to discover the dynamics of so-called "light bullets" (LB), i.e. optical pulses with relatively equal transfer and longitudinal size. Our investigation corrects this idea and finds that the equation (12) governed the propagation of optical pulses with longitudinal size hundred times greater than the transfer one in UV region of dielectrics and liquids. In the case of optical waves with very small intensity the nonlinear term can be neglected $\gamma \ll 1$, and we obtain the linear version of the equation (12):

$$-i\alpha\delta^2 \frac{\partial \vec{A}}{\partial z} + \frac{1}{2} \Delta_{\perp} \vec{A} - \frac{1}{2} \beta_1 \frac{\partial^2 \vec{A}}{\partial t^2} = 0. \tag{13}$$

Let investigate the two component vector field polarizable in transverse x, y plane. For long pulses, as they are more close to the cw regime, the longitudinal vector component is small and such approximation is possible. We represent the optical vector field by two orthogonal components $A_1 \vec{x} + A_2 \vec{y}$ and the equation (12) is transformed to the next nonlinear system of equations:

$$\begin{aligned}
& -i\alpha\delta^2\frac{\partial A_1}{\partial z} + \frac{1}{2}\Delta_{\perp}A_1 - \frac{1}{2}\beta_1\frac{\partial^2 A_1}{\partial t^2} + \gamma(|A_1|^2 + |A_2|^2)A_1 = 0, \\
& -i\alpha\delta^2\frac{\partial A_2}{\partial z} + \frac{1}{2}\Delta_{\perp}A_2 - \frac{1}{2}\beta_1\frac{\partial^2 A_2}{\partial t^2} + \gamma(|A_2|^2 + |A_1|^2)A_2 = 0.
\end{aligned} \tag{14}$$

In the linear case the (14) is reduced to two equal scalar equations for A_1 and A_2 :

$$-i\alpha\delta^2\frac{\partial A_i}{\partial z} + \frac{1}{2}\Delta_{\perp}A_i - \frac{1}{2}\beta_1\frac{\partial^2 A_i}{\partial t^2} = 0, \tag{15}$$

where $i = 1, 2$. We rewrite the linear scalar equation (15) in cylindrical coordinates:

$$-i\alpha\delta^2\frac{\partial A_i}{\partial z} + \frac{1}{2}\left(\frac{1}{r}\frac{\partial A_i}{\partial r} + \frac{\partial^2 A_i}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 A_i}{\partial \theta^2}\right) - \frac{1}{2}\beta_1\frac{\partial^2 A_i}{\partial t^2} = 0, \tag{16}$$

where $r = \sqrt{x^2 + y^2}$ and $\theta = \arctan(x/y)$. The linear equation (16) admits infinite number of exact analytical solutions of kind of:

$$A_i = r^n [a_n \cos(n\theta) + a_n \sin(n\theta)] \exp\left(i(\beta_1 z + \sqrt{2\alpha\delta^2}t)\right), \tag{17}$$

$$A_i = r^{-n} [a_n \cos(n\theta) + a_n \sin(n\theta)] \exp\left(i(\beta_1 z + \sqrt{2\alpha\delta^2}t)\right), \tag{18}$$

where $n = 0, 1, 2, \dots, \infty$ is one infinite number. The solutions (17), (18) admit modulation frequency $\Omega = \sqrt{2\alpha\delta^2}$ and modulation wave vector $K = \beta_1$ in z direction. The existence of such kind solutions for monochromatic light was observed for first time by Nye and Berry [14]. The solutions of kind (17), (18) admit amplitude and phase singularities. One elegant way to remove the amplitude singularities by using the combination of Gaussian envelope and the solution (18) with $n = 1$ was suggested for first time in [15] for paraxial beam. This method is applicable also to the linear equation (16) and optical vortices in combined pulses with finite amplitudes can be found. Usually the

localized solutions of the linear equations are not stable. Now we will turn to most interesting case; exact solutions of the nonlinear system of equation (14). Again we rewrite the system (14) in cylindrical variables:

$$\begin{aligned}
& -i\alpha\delta^2\frac{\partial A_1}{\partial z} + \frac{1}{2}\left(\frac{1}{r}\frac{\partial A_1}{\partial r} + \frac{\partial^2 A_1}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 A_1}{\partial \theta^2}\right) \\
& -\frac{1}{2}\beta_1\frac{\partial^2 A_1}{\partial t^2} + \gamma(|A_1|^2 + |A_2|^2)A_1 = 0, \\
& -i\alpha\delta^2\frac{\partial A_2}{\partial z} + \frac{1}{2}\left(\frac{1}{r}\frac{\partial A_2}{\partial r} + \frac{\partial^2 A_2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 A_2}{\partial \theta^2}\right) \\
& -\frac{1}{2}\beta_1\frac{\partial^2 A_2}{\partial t^2} + \gamma(|A_2|^2 + |A_1|^2)A_2 = 0.
\end{aligned} \tag{19}$$

The nonlinear system of equations (19) admits another infinite number of exact vortex solutions of kind of:

$$A_1 = \sqrt{((n+1)^2 - 1)/\gamma(r^{-1})} \cos((n+1)\theta) \exp\left(i(\beta_1 z + \sqrt{2\alpha\delta^2}t)\right), \tag{20}$$

$$A_2 = \sqrt{((n+1)^2 - 1)/\gamma(r^{-1})} \sin((n+1)\theta) \exp\left(i(\beta_1 z + \sqrt{2\alpha\delta^2}t)\right), \tag{21}$$

where $n = 1, 2, \dots, \infty$ is one infinite number. One important consequence of these solutions is, that they are obtained as a balance not only between the diffraction and the nonlinearity, but mostly by the nonlinearity and the angular distribution. That is why, we can expect one stable propagation of such vortices in z direction. Right away we can point here, that conditions for finiteness of the energy of the above solutions have been not found up to now.

4 Pulses with equal transverse and longitudinal size $r_\perp \sim z_0$

The investigation of the propagation of optical pulses or modulated waves with equal transverse and longitudinal size $r_\perp \sim z_0$ is more convenient to provide in coordinate system moving with group velocity. That is why we apply a Galilean transformation to the basic vector amplitude equation ($t' =$

$t, z' = z - vt$), as difference from the case of long pulses, where the "moving in time" transformation is more natural. The equation (1) in this case is transformed to:

$$\begin{aligned}
& -i \left(\frac{\partial \vec{A}}{\partial t'} + \left(n_2 + \frac{k_0 v}{2} \frac{\partial n_2}{\partial \omega} \right) \left(\frac{\partial \left(|\vec{A}|^2 \vec{A} \right)}{\partial t'} - v \frac{\partial \left(|\vec{A}|^2 \vec{A} \right)}{\partial z'} \right) \right) + \\
& \frac{v}{2k_0} \Delta_{\perp} \vec{A} - \frac{v^3 k_0''}{2} \frac{\partial^2 \vec{A}}{\partial z'^2} - \frac{v}{2} \left(k_0'' + \frac{1}{k_0 v^2} \right) \left(\frac{\partial^2 \vec{A}}{\partial t'^2} - 2v \frac{\partial^2 \vec{A}}{\partial t' \partial z'} \right) + \quad (22) \\
& \frac{n_2 k_0 v}{2} |\vec{A}|^2 \vec{A} = 0,
\end{aligned}$$

where $\Delta_{\perp} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. We use again the approximation $\partial n_2 / \partial \omega \simeq 0$. To estimate the influence of the different terms on the propagation dynamics we rewrite the equation (22) in dimensionless form. Defining the rescaled variables:

$$\vec{A} = A_0 \vec{A}'; \quad x = r_0 x'; \quad y = r_0 y'; \quad z'' = r_0 z'; \quad t'' = t_0 t', \quad (23)$$

and constants:

$$\alpha = 2k_0 r_0^2 / t_0 v = 2k_0 r_0; \quad \beta = v^2 k'' k_0; \quad \gamma = k_0^2 r_0^2 n_2 |A_0|^2; \quad (24)$$

$$r_0 / t_0 = v; \quad \gamma_1 = 2k_0 r_0 n_2 |A_0|^2,$$

equation (22) can be transformed in the following (the primes and the seconds are not written for clarity again):

$$\begin{aligned}
& -i \left(\alpha \frac{\partial \vec{A}}{\partial t} + \gamma_1 \left(\frac{\partial \left(|\vec{A}|^2 \vec{A} \right)}{\partial t} - \frac{\partial \left(|\vec{A}|^2 \vec{A} \right)}{\partial z} \right) \right) + \Delta_{\perp} \vec{A} - \beta \frac{\partial^2 \vec{A}}{\partial z^2} - \\
& (\beta + 1) \left(\frac{\partial^2 \vec{A}}{\partial t^2} - 2 \frac{\partial^2 \vec{A}}{\partial t \partial z} \right) + \gamma |\vec{A}|^2 \vec{A} = 0. \quad (25)
\end{aligned}$$

We point out, that the normalizing is with equal longitudinal and transverse dimensions r_0 . As it was pointed above, it is fulfilled usually for few picosecond and femtosecond pulses of a pulse with transverse size of 1–5 millimeters to hundred micrometers.

4.1 Equation in optical region of media with weak dispersion $\beta \ll 1$

Dimensionless dispersion parameter β in transparency region of dielectrics and gases usually is of the order of $10^{-2} - 10^{-3}$. For example we calculate β in the transparency region of pure silica on $\lambda = 1,55\mu m$ using the standard Sellmeier expression for dependence of the dielectric constant from frequency, and the value really is of this order $\beta = -0,0069$. The constant α has a value of $\alpha \approx 10^2$ ($\alpha \approx 2r_0k_0$) if the slowly varying approximation is used. In this case, when we investigate the optical pulses with power near to the critical for self-focusing $\gamma \approx 1$, $\gamma_1 \ll 1$, our vector amplitude equation (25) can be transformed to the next evolution equation:

$$-i\alpha\frac{\partial\vec{A}}{\partial t} + \Delta_{\perp}\vec{A} - \left(\frac{\partial^2\vec{A}}{\partial t^2} - 2\frac{\partial^2\vec{A}}{\partial t\partial z}\right) + \gamma|\vec{A}|^2\vec{A} = 0. \quad (26)$$

The equation (26) is one generalization of the well known paraxial equation (7) in optical region for pulses with equal transverse and longitudinal size.

4.2 Equation in UV transparency region. Strong dispersion ($\beta \approx 1$)

The critical dimensionless parameter β , which determines the relation between dispersion and diffraction in the normalized amplitude equation (25), as it was established in the previous paragraph, is very little in the optical region. In spite of this, the temporal effects are presented in the corresponding equation (26) and for pulses with equal transverse and longitudinal size can not be neglected. The situation in UV region is quite different. In this region we can reach $\beta \simeq 1$. When we investigate optical pulses with power, near to the critical for self-focusing, the corresponding nonlinear dimensionless parameters admit values $\gamma \approx 1$, $\gamma_1 \ll 1$. Then the equation (25) is transformed to:

$$-i\alpha \frac{\partial \vec{A}}{\partial t} + \Delta_{\perp} \vec{A} - \beta \frac{\partial^2 \vec{A}}{\partial z^2} - (\beta + 1) \left(\frac{\partial^2 \vec{A}}{\partial t^2} - 2 \frac{\partial^2 \vec{A}}{\partial t \partial z} \right) + \gamma |\vec{A}|^2 \vec{A} = 0. \quad (27)$$

If we compare the equations in optical (26) and UV region (27), it is seen that in UV region we can take in mind in addition a negative second derivative in z direction of amplitude function and we multiply the temporal effects of second order by factor $\beta + 1$. We use for example again bulk fused silica and calculate the linear refractive index $n^2(\omega)$ from the Sellmeier relations. The investigation is provided for the spectral region from wavelength $\lambda = 0.264 \mu m$ to wavelength $\lambda = 0.164 \mu m$. The group velocity v_g , dispersion $k''(\omega)$ and the dimensionless parameter $\beta(\omega)$ are calculated. The typical characteristic of the laser pulse are $t_0 = 5 \times 10^{-12} s$, group velocity $v_g \simeq 1.8 \times 10^{10} cm/s$ and $r_{\perp} = z_0 \simeq 1 \times 10^{-1} cm$. On Figure 1, Figure 2 are presented correspondingly the graphics of dispersion of the group velocity $k''(\lambda)$ and dimensionless parameter $\beta(\lambda)$. As it is seen from Fig.2, the dimensionless parameter β , really can reach values equal to one in UV region for some dielectrics.

4.3 Equation in media with strong negative dispersion $\beta \approx -1$

Now we come to the most interesting case in this investigation: the possibility that the temporal effects of second order can be neglected. As it can be traced out from basic amplitude equation (1), and to see in the normalized equation (25), when $k_0 v_g^2 k'' = \beta \cong -1$, the dispersion effects of second order do not present in the corresponding amplitude equations and do not influence on the propagation dynamics. Recently in [16] it was found that near the electronic resonances of gases and dielectrics and also near the Langmuir frequency in cold plasma the dispersion is negative, the dispersion parameter arises rapidly and is of the order of $\beta \approx -1$. Again we investigate the propagation of optical pulses with power near the critical for self-focusing ($\gamma \approx 1$, $\gamma_1 \ll 1$). The amplitude equation (25) in this case can be written in the next simple form of 3D+1 Vector Nonlinear Schrödinger equation (VNSE):

$$-i\alpha \frac{\partial \vec{A}}{\partial t} + \Delta_{\perp} \vec{A} + \frac{\partial^2 \vec{A}}{\partial z^2} + |\vec{A}|^2 \vec{A} = 0. \quad (28)$$

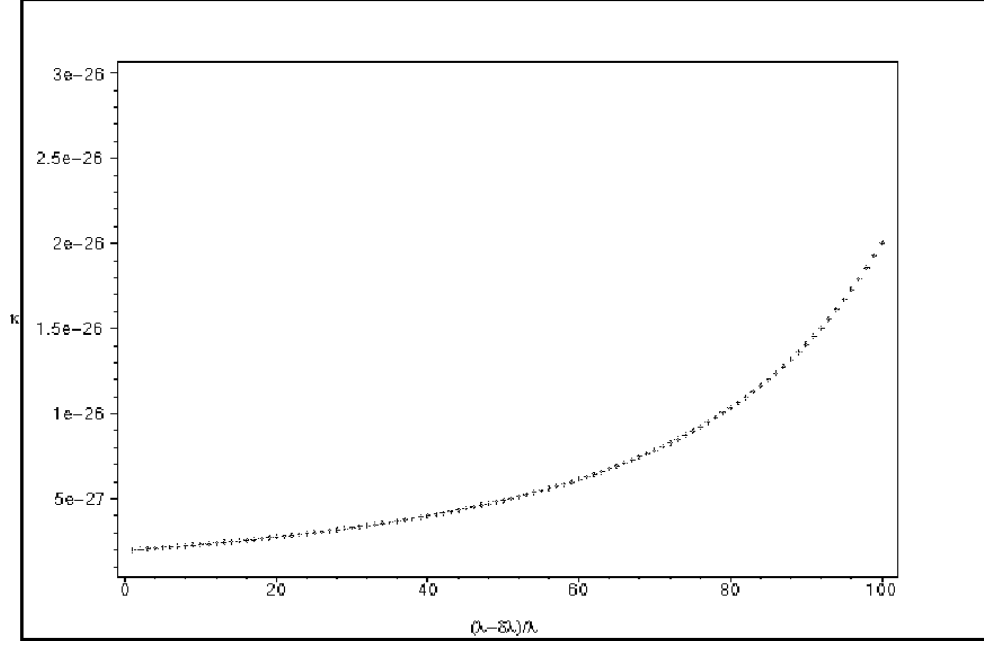


Figure 1: Plot of the dispersion of the group velocity $k''(\omega)$ in the UV region from $\lambda = 264$ nm to $\lambda = 164$ nm for bulk fused silica.

Here we will study the possibility to obtain $\beta = -1$ in the next three cases: 1. Near the isolated resonance of mercury vapor on $\lambda = 0.3653\mu m$. 2. Near the Langmuir frequency in cold electron plasma. 3. In deep UV and R  region for dielectrics, gazes and metals.

4.3.1 Dispersion parameter β near the isolated electron resonance

When $\varepsilon(\omega) \sim 1/\omega$ the wave vector can be written as:

$$k(\omega) = \frac{\sqrt{\omega}}{c}. \quad (29)$$

Calculating $\beta = v^2 k'' k$ for dispersion relation of kind of (29), right away we obtain $\beta = -1$. This gives us a confidence to look for such zones near the electronic resonances where $\varepsilon(\omega) \sim 1/\omega$. Near to some of the isolated electron resonances of metal vapors the dielectric constant can be expressed by [24]:

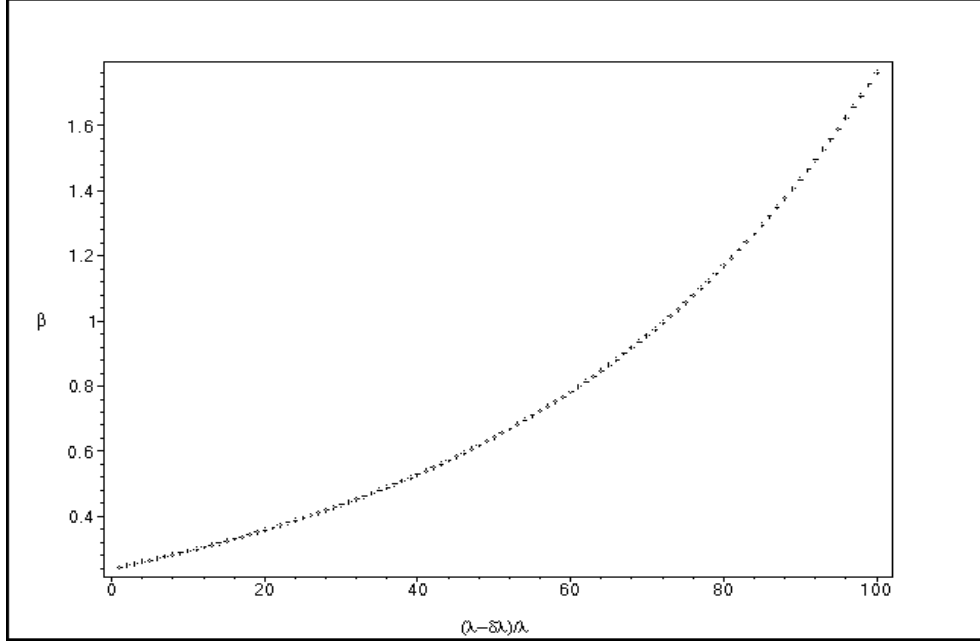


Figure 2: Plot of the dimensionless parameter $\beta = k_0 v^2 k''$ within wavelengths from $\lambda = 264$ nm to $\lambda = 164$ nm for bulk fused silica. The dispersion parameter reach a value $\beta \sim 1$ in UV region.

$$\varepsilon(\omega) = 1 + \frac{f4\pi N e^2}{m\omega_0} \frac{\omega_0 - \omega}{(\omega_0 - \omega)^2 + \gamma^2}. \quad (30)$$

The investigation are provided for resonance frequency of mercury vapor on $\omega_0 = 2\pi c/\lambda_0 = 0.515568 \cdot 10^{16}$ Hz, oscillator strength $f = 0.3$, and natural (i.e. radiative) linewidth $\gamma = 1/4, 53\lambda_0^2 = 1,654 \cdot 10^8$ Hz. We use Lorenz shape of the line, as we investigate the rarefied vapor and looking in zones outlying from resonance on distance more than 4γ , where the natural shape of the line dominated. In Figure 3 it is shown the form of the dielectric constant for number density $N = 1,2 \cdot 10^{15}$. In the Figure 4 it is presented the dimensionless dispersion parameter β . As it is well seen on the wing on the resonance on distance about 4γ we have spectral zone where $\beta \simeq -1$. Right away it must be pointed that β depends strongly on the number density N . The number density of the order of $N \approx 1 \cdot 10^{13} - 1 \cdot 10^{15}$ appears to be

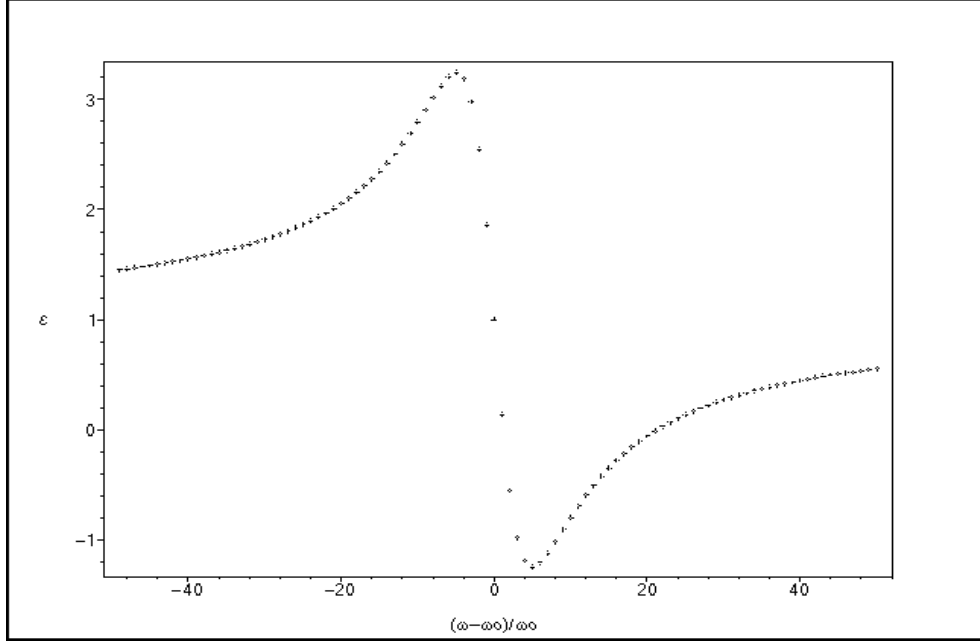


Figure 3: Frequency dependence of the dielectric constant for resonance frequency of mercury vapor of $\omega_0 = 2\pi c/\lambda_0 = 0.515568 \cdot 10^{16}$ Hz ($\lambda = 365$ nm), and natural linewidth $\gamma = 1/4, 53\lambda_0^2 = 1,654 \cdot 10^8$ Hz.

most suitable for large zones with $\beta \sim -1$. Seems, for such number density the wing of the resonance in best way can be approximated with $\varepsilon(\omega) \sim 1/\omega$.

4.3.2 Dispersion parameter β near to Langmuir frequency in cold plasma

Dielectric constant in the case of cold electron plasma is given by the expression [25]:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \quad (31)$$

where $\omega_p = \sqrt{4\pi N_1 e^2/m}$ is the plasma frequency and N_1 is the number of free electron particles per cm^{-3} . Here we investigate a typical number which characterized the plasma density in generators used for nuclear fusion. In Figure

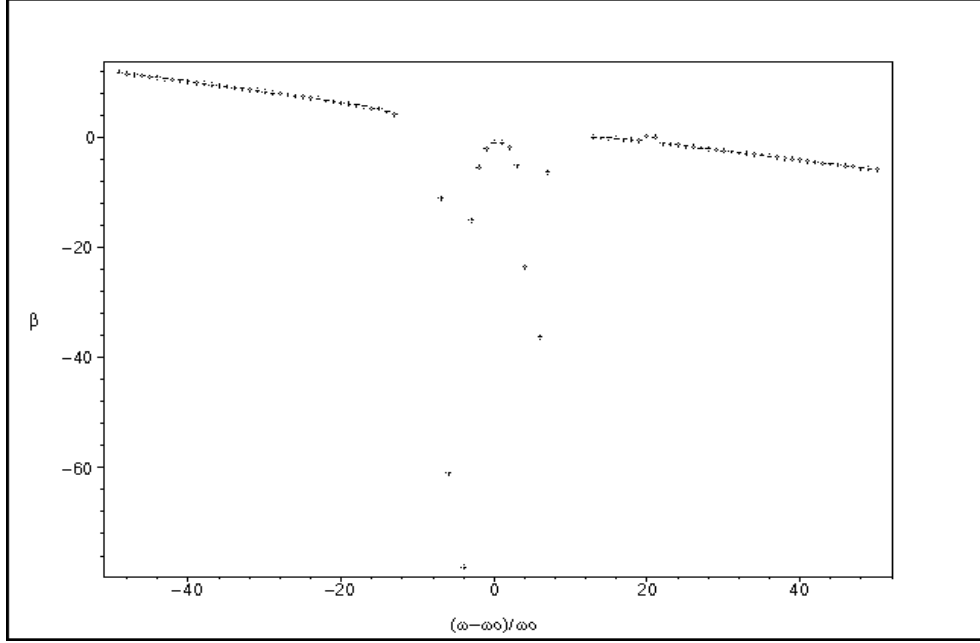


Figure 4: Plot of the dimensionless parameter $\beta = k_0 v^2 k''$ near to resonance frequency of mercury vapor on $\omega_0 = 2\pi c/\lambda_0 = 0.515568 \cdot 10^{16}$ Hz. On distance 4γ we find the spectral zone where $\beta \simeq -1$.

5 is presented $\varepsilon(\omega)$ for frequencies higher and close to plasma frequency and number density $N_1 \cong 1 \cdot 10^{17} \text{ cm}^{-3}$. The frequency is normalizing on spectral width of optical pulse $\delta\omega = 1 \cdot 10^{10}$ Hz, which corresponds to pulse with time duration $\Delta t \simeq 2 \cdot 10^{-10}$ seconds. The corresponding to this dielectric constant dispersion parameter $\beta(\omega)$ is presented in Figure 6. As it is seen from the graphics, the large spectral zone exists, more than twenty spectral widths from plasma frequency, where $\beta \cong -1$. The transverse and longitudinal size such optical pulse valued from $0.5 - 1 \text{ cm}$. An important relation is pointed out with investigation of dependence of β from time duration of the pulses and number density. With decreasing the time and spatial duration of the pulses (fs pulses) and increasing the number density, this large spectral zone with $\beta \cong -1$ keeps on.

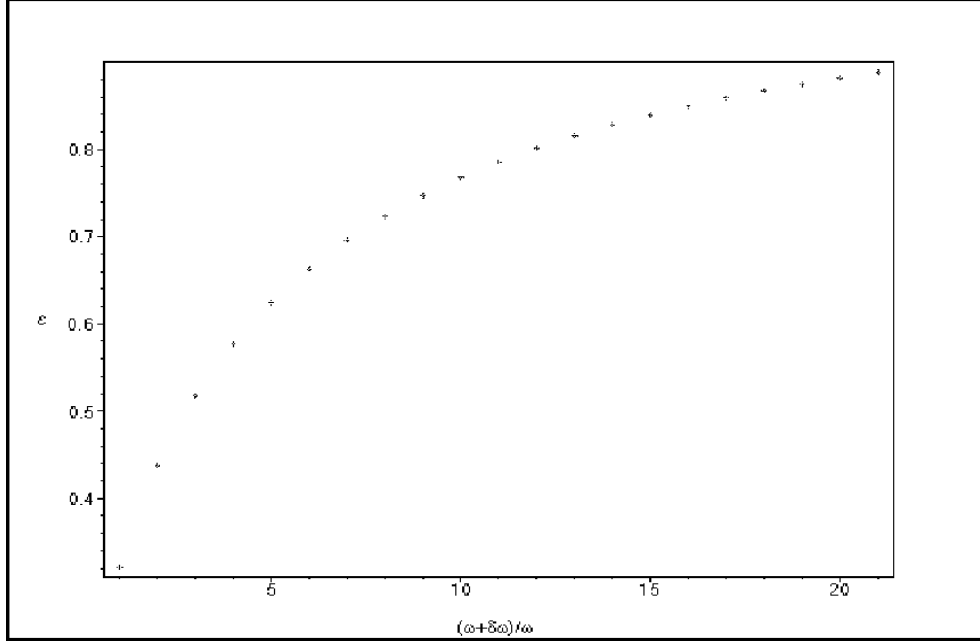


Figure 5: Frequency dependence of the dielectric constant near to Langmuir frequency in cold plasma for number density $N_1 \sim 1.10^{17}$. The frequency is normalizing on spectral width of the optical pulse $\delta\omega = 1.10^{10}$ Hz.

4.3.3 Dielectric constant $\epsilon(\omega)$ and dispersion parameter β from deep UV to R  region

As it was established in [26], it is possible to find equal dispersion relation of the dielectric constant $\epsilon(\omega)$ for different kind of materials, as dielectrics, gases, semiconductors, metals, when the carrying frequency of optical wave is much higher than the resonances of this media. In these cases the localized energy of optical wave is much higher than the critical for ionization of the materials and the linear and nonlinear response is the same as in cold plasma. Practically the electrons interact as free electrons on these frequencies. It is not hard to find that in this region again we have the dispersion relation of dielectric constant of the kind of:

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}. \quad (32)$$

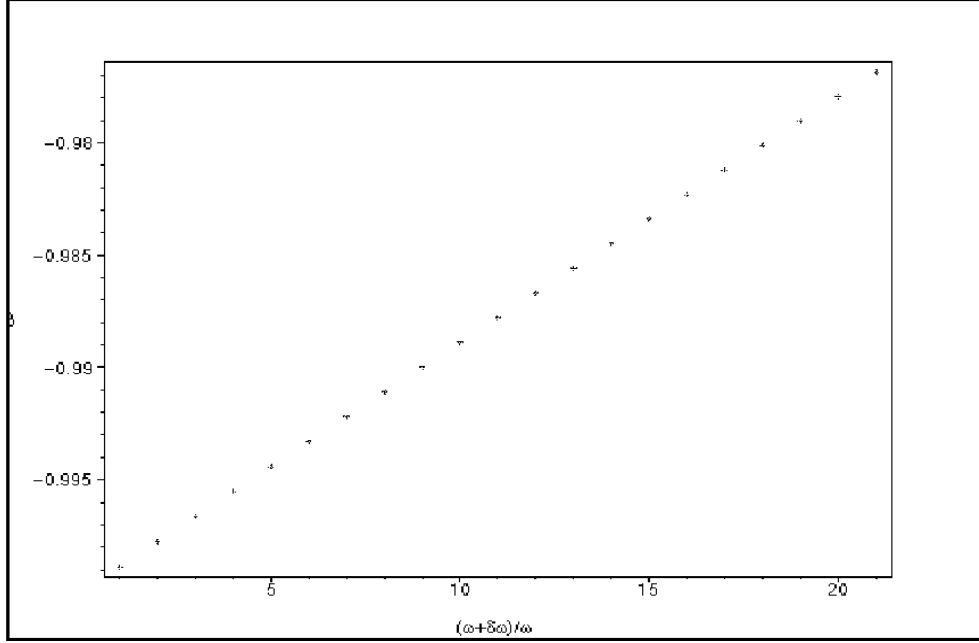


Figure 6: The corresponding dimensionless dispersion parameter β near the plasma frequency. Large spectral zone with $\beta \simeq -1$ is established.

The critical parameter which characterized the plasma frequency in these materials is the number density N_1 of the free electrons. The typical number for semiconductors is $N_1 \cong 1.10^{18}$ and for metals is $N_1 \cong 1.10^{22}$. That is why, the application of this formula started from deep UV for light elements as H, Li, e.c. and from Rö frequencies in heavy elements. For optical waves when $\omega > \omega_p$, there are no difference between metals, semi-conductors, dielectrics and metals. All of them are transparent for optical waves with carrying frequency $\omega_0 > \omega_p$. Again we can obtain, that near the corresponding plasma frequency of these materials, large frequency zone with negative dispersion parameter of order of $\beta \sim -1$ exists, if the picosecond and femtosecond pulses are used. For all of these three important cases the propagation of localized optical pulses is governed by the vector 3D+1 Nonlinear Schrödinger Equation (28). The numerical experiment provided in [16] show a stable propagation of the vortex solutions. The intensity picture of these multi-component solutions is truly symmetric and look like as stable Fraunhofer distribution. The internal structure of these vortices is presented by the

spherical harmonics with integer number.

5 Exact vortex solutions on VNSE

The solutions of the 3D+1 Vector Nonlinear Schrödinger amplitude equation (28) in a fixed basis are [16]:

$$A_x = \sqrt{2} \frac{\exp(i\sqrt{\alpha\Omega}r)}{r} \sin\theta \cos\varphi \exp(i\Omega t), \quad (33)$$

$$A_y = \sqrt{2} \frac{\exp(i\sqrt{\alpha\Omega}r)}{r} \sin\theta \sin\varphi \exp(i\Omega t), \quad (34)$$

$$A_z = \sqrt{2} \frac{\exp(i\sqrt{\alpha\Omega}r)}{r} \cos\theta \exp(i\Omega t), \quad (35)$$

where $r = \sqrt{x^2 + y^2 + z^2}$, $\theta = \arccos \frac{z}{r}$ and $\varphi = \arctan \frac{x}{y}$ are the moving spherical variables of the independent variables x, y, z . The corresponding real solutions $\Re(\vec{A})$, are:

$$\Re(A_x) = \frac{1}{2i} (A_x - A_x^*) = \sqrt{2} \frac{\sin(\sqrt{\alpha\Omega}r + \Omega t)}{r} \sin\theta \cos\varphi, \quad (36)$$

$$\Re(A_y) = \frac{1}{2i} (A_y - A_y^*) = \sqrt{2} \frac{\sin(\sqrt{\alpha\Omega}r + \Omega t)}{r} \sin\theta \sin\varphi, \quad (37)$$

$$\Re(A_z) = \frac{1}{2i} (A_z - A_z^*) = \sqrt{2} \frac{\sin(\sqrt{\alpha\Omega}r + \Omega t)}{r} \cos\theta. \quad (38)$$

It is important to point, that in [16] conditions for finiteness of the energy of the solutions (36)-(38) was found. As it can be established, the averaged

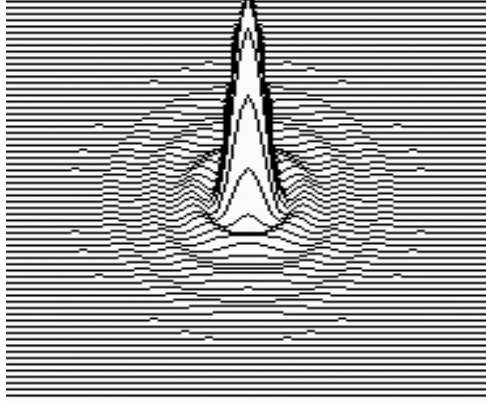


Figure 7: Fraunhofer distribution for the averaged with one time period intensity of the solutions (39) of 3D+1 Vector Nonlinear Schrödinger equation (28).

with one time period intensity field of these solutions admits total radial symmetry:

$$\langle A_x \rangle^2 + \langle A_y \rangle^2 + \langle A_z \rangle^2 = 2 \frac{\sin^2(\sqrt{\alpha\Omega}r)}{r^2}. \quad (39)$$

Thus, in one experiment the solution will look like as Fraunhofer distribution. Figure 7 presents the surface $\langle \vec{A} \rangle_{z=0}^2$ of the solutions (36)-(38).

We can recognize the vortex structures of these solutions only if we see the field distribution and the intensity of one of the component, for example $\langle A_x \rangle$. On Figure 8 we present the surface of one of the components $\langle A_{x,|z=0} \rangle$, rewritten in Cartesian coordinates:

$$\langle A_x \rangle = \sqrt{2} \frac{x \sin(\sqrt{x^2 + y^2 + z^2})}{x^2 + y^2 + z^2}. \quad (40)$$

It is clearly seen, that the surface of our three dimensional solutions possess linear phase dislocations in space, which characterizes the vortex structures [14].

The intensity of this component $\langle A_x^2 \rangle$, which is one $l = 1, m = 1$ distribution, is presented in Figure 9. This picture is possible to see in one

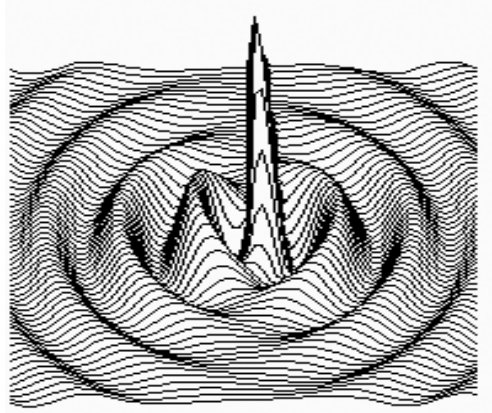


Figure 8: Surface of the one of the component $\langle A_{x,|z=0} \rangle$ of the averaged with one time period vortex solutions (40). It is clearly seen linear phase dislocation for these solutions.

experiment, when an optical vortex of kind (36)-(38) is passed through the linear polarizer. Then the internal structure of these vortex solutions composed by spherical harmonics with $l = 1, m = \pm 1, 0$ distribution, will be seen. Our solutions (36)-(38) admit total angular momentum $l = 1$ and line phase singularities for the different components. That is why, in spite of the fact that the intensity field is total symmetric, we recognize the internal vortex structures of these solutions.

6 Conclusion

In conclusion, we investigated the application of the nonlinear amplitude equation (1), in different spectral zones, starting from the optical region, and extending up to the Rö region of dielectrics. Different generalizations of the paraxial equation are obtained, depending from the spectral zones, the material constants, and the initial shape of the pulses (long pulses or optical bullets). We find large spectral region near the plasma frequency in cold plasma for fs and few picosecond optical pulses, where the temporal effects of second order in the amplitude equations (1) can be neglected. The same result is obtained also for spectral regions on the 'wings' of the electron resonances in metal vapors. In the last case the conditions to obtain such

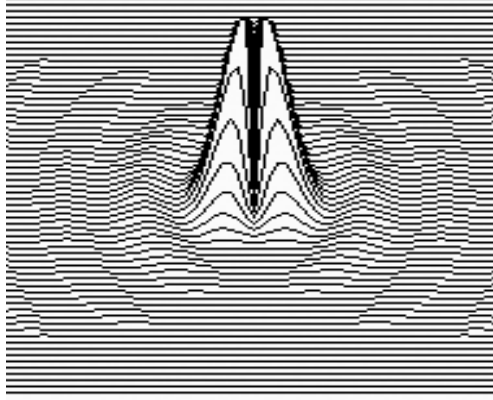


Figure 9: The intensity profile of one of the components $\langle A_{x,|z=0} \rangle^2$, which corresponds to $l = 1, m = 1$ distribution.

effects depend strongly on the number density of the gases. In these spectral zones the propagation of optical pulses is governed by the amplitude vector nonlinear Schrödinger equation (28) and, as it was predicted in [16], stable 3D optical vortices can be observed. They look like as Fraunhofer distribution in the space. After passing through the linear polarizer the vortex structures will be seen. The vector theory presented in this paper can be extended also in respect to different kind of nonlinearities and polarizations of the optical field.

7 Appendix 1. Nonlinear polarization of two and three components of the same frequency

The electric field associated with linear or elliptically polarized optical wave can be written in the form

$$\vec{E}(x, y, z, t) = \frac{1}{2i} ((\vec{x}A_x + \vec{y}A_y) \exp(i\omega_0 t) - c.c.), \quad (41)$$

where A_x and A_y are the complex amplitudes of the polarization components of a wave with the frequency ω_0 .

7.1 Linearly polarized components

Let investigate the polarization dynamics of two initially linearly polarized components of the electrical field. Then, there is no initial phase different between the components, and the complex amplitudes can be expressed as a product of two real amplitude functions with equal phases: $\Re(\hat{A}_x) \exp(i\phi)$, $\Re(\hat{A}_y) \exp(i\phi)$. Calculating the relation between the linear polarized components in the same manner, as it was performed for plane wave [27], we obtain the same kind relation for the components of the electrical field:

$$\frac{E_y}{E_x} = \frac{\Re(\hat{A}_y)}{\Re(\hat{A}_x)}, \quad (42)$$

where $\Re(\hat{A}_y)$ and $\Re(\hat{A}_x)$ are real functions, not real constants as in the case of plane wave. We investigate the propagation dynamics of optical pulses with arbitrary localized amplitudes. The soliton case appear as partial case, when equal amount of nonlinear and linear phase shift are compensated, the initial phase different do not changed, and the linear polarization keeps on. For optical pulses with arbitrary amplitudes, as the media is isotropic, the diffraction and the dispersion add equal phases to the two components and they can be written generally in the form:

$$A_x = \Re(A_x) \exp(i(K_{diff}z + \Omega_{disp}t) + \phi) = \Re(A_x) \exp(i\Phi(z, t)), \quad (43)$$

$$A_y = \Re(A_y) \exp(i(K_{diff}z + \Omega_{disp}t) + \phi) = \Re(A_y) \exp(i\Phi(z, t)), \quad (44)$$

where K_{diff} is the phase shift due to diffraction, Ω_{disp} is the phase shift due to dispersion, $\Phi(z, t)$ is the equal phase components for the both amplitudes, and $\Re(A_y)$, $\Re(A_x)$ are the real part of solutions for the amplitudes on distance z in isotropic media. For arbitrary solutions of kind (43), (44) the relation (42) keeps on for the real part of the amplitudes. It is following from the fact, that the diffraction and the dispersion effects do not change the initial linear polarization of the pulses (do not yield linear birefringence in isotropic materials). The nonlinear polarization \vec{P}_{nl} in the case of isotropic medium is well known and can be written in the form [28, 26]:

$$\vec{P}_{nl} = A(\omega) (\vec{E} \cdot \vec{E}^*) \vec{E} + \frac{B(\omega)}{2} (\vec{E} \cdot \vec{E}) \vec{E}^*. \quad (45)$$

Substituting the expression for the two linearly polarized component (43) and (44) in (45), right way we obtain for the nonlinear polarization of the different components [26]:

$$P_{nl}^x = \left(A(\omega) + \frac{B(\omega)}{2} \right) (|A_x|^2 + |A_y|^2) A_x, \quad (46)$$

$$P_{nl}^y = \left(A(\omega) + \frac{B(\omega)}{2} \right) (|A_x|^2 + |A_y|^2) A_y. \quad (47)$$

We point here, that the expression is valid not only for nonlinear nonresonant optical response, when $A = B = \text{const}$, but also for nonlinearity near to some of the resonances, when $A(\omega) \neq B(\omega)$. In the same way we can obtain, that the components of the nonlinear polarization for one linear polarized vector in 3D space take forms:

$$P_{nl}^x = \left(A(\omega) + \frac{B(\omega)}{2} \right) (|A_x|^2 + |A_y|^2 + |A_z|^2) A_x, \quad (48)$$

$$P_{nl}^y = \left(A(\omega) + \frac{B(\omega)}{2} \right) (|A_x|^2 + |A_y|^2 + |A_z|^2) A_y, \quad (49)$$

$$P_{nl}^z = \left(A(\omega) + \frac{B(\omega)}{2} \right) (|A_x|^2 + |A_y|^2 + |A_z|^2) A_z. \quad (50)$$

7.2 Elliptically polarized components

In the case of elliptically polarized light the relation (42) take form:

$$\frac{E_y}{E_x} = \frac{\Re(A_y)}{\Re(A_x)} \exp(i(\delta_y - \delta_x)), \quad (51)$$

where $\delta_y - \delta_x \neq 0$, $n\pi$, $\pi/2$, is the different of the phase functions for the components of the vector field. Since the diffraction and the dispersion add equal amount of phase shift to the different components in isotropic media, the phase different keeps on again. If we substitute E_x and E_y with such phase difference in the expression for nonlinear polarization (45), and take in mind that $A = B = \text{const}$ for case of nonresonant electronic nonlinearities, the next expression for the components can be found [29]:

$$P_{nl}^x = \text{const} \left[\left(|A_x|^2 + \frac{2}{3} |A_y|^2 \right) A_x + \frac{1}{3} (A_x^* A_y) A_y \exp(2i(\delta_x - \delta_y)) \right], \quad (52)$$

$$P_{nl}^y = \text{const} \left[\left(|A_y|^2 + \frac{2}{3} |A_x|^2 \right) A_y + \frac{1}{3} (A_y^* A_x) A_x \exp(-2i(\delta_x - \delta_y)) \right]. \quad (53)$$

The last terms in equations (52) and (53) present degenerate four wave mixing process ($\omega_1 = \omega_2 = \omega_0$). Since $\delta_x - \delta_y \neq 0$, this leads to periodically exchange of energy between the elliptically polarized components. Thus, such one exchange of energy can be observed in isotropic material as gases, metal vapors and crystals. When the phase different vanish ($\delta_x - \delta_y = 0$) in the case of linearly polarized components, the amplitude functions admit again equal phases, and the nonlinear polarization (52) and (53) can be transformed to the nonlinear polarization of linearly polarized components (46) and (47). This is one generalization of the results obtained for parametric solitons in $\chi^{(3)}$ media [30]. When there is no phase different between the components, the parametric term appear as usual cross-phase term in the dispersion relations, and can be take in mind to determinate the right amplitude constants. The polarization dynamics in materials which admit modal birefringence is more complicate. In single-mode fiber, where the linear birefringent effect appear [31], additional polarization mode dispersion exists [32]. This lead to random birefringence in optical fibers and random phase different in the last parametric terms in (52) and (53). Our numerical experiment show that the parametric effects with random phases are not effective, and can be neglected. One transitions to the Manakov system with coefficients equal to one in front of the cross-terms [21] is possible again for birefringent materials at elliptically angle 35° between the main axes [33].

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